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Simulation of surface waves generation by an underwater landslide

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Abstract — The present paper discusses the results of the numerical simulation of the process of surface wave generation by a moving underwater landslide. The computational algorithms are based on finite difference schemes for shallow water equations of different orders of hydrodynamic approximation and equations for potential ideal fluid flows with a free boundary. The dependence of the parameters of generated waves on the law of motion of the migrating bottom fragment is studied.

The interest in the problems of wave generation by underwater landslides is due to persistent attempts to relate the known facts of the occurrence of anomalous tsunami waves to the sliding mechanism of their generation, as opposed to the conventional seismic mechanism [16, 17]. Here anomaly implies an inconsistency between a weak seismic event and a significant tsunami wave near the coast. In the past years such phenomena have been recorded near the coasts of Canada, Turkey and Papua New Guinea.

Under natural conditions, an underwater landslide represents the motion of a mass of material down the slope of the bottom. The large volume of the moving mass induces waves on the water surface, which are close in their characteristics to waves caused by a tsunami-induced earthquake.

A hypothesis put forward in the 1930s states that even a weak earthquake can cause the motion of considerable sliding masses in the coastal zone, whose origin is explained by the accumulation of alluvia carried by rivers, avalanches from the nearest heights, etc. There are situations where these masses turn out to be fully or partially flooded. In the latter case, the wave generation process proves practically simultaneous with the process of their running up the beach.

The known approaches to the simulation of landslides include the simulation of the motion of an absolutely rigid body [6, 16, 17] or a set of such bodies [15], simulating a fluid flow of different density, viscosity, etc. [8, 10, 14], or simulating the motion of a plastoelastic medium [5] with or without taking into account the interaction with the ambient fluid. In some situations, it appears promising to simulate the phenomenon as a two-layer fluid with layers of various densities and viscosity

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coefficients [7, 9, 11]. The specifics of the simulation of respective surface waves is that the tsunami waves of the sliding origin are generated in the coastal zone of a small depth, the duration of the landslide motion is quite long and comparable with the period of the generated wave; the characteristic depth and the vertical size of the landslide are comparable too.

The aim of the present paper is to develop and study the hydrodynamic component of a consistent mathematical model of earth motion and waves transformation on a free surface. The mathematical modelling of these wave regimes is implemented by an hierarchy of wave hydrodynamics models comprising the equations of shallow water theory in approximations that take into account nonlinear and dispersion effects and full equations for ideal fluid wave hydrodynamics. This makes it possible to solve one of the main problems of mathematical modelling, i.e., specify the domains of the adequacy of mathematical models by comparison with data of laboratory experiments. In this paper, comparison was made with the results of the experiments [3] determining the parameters of the wave regime caused by the motion of a fully submerged body down the slope.

For hyperbolic equations we used simple cost-effictive and finite-difference algorithms based on second-order approximation shemes, which comprise a number of parameters that allow us to control the contribution of the nonlinear and dispersion effects and use the smoothing procedure selectively. To approximate the full hydrodynamic model we have used schemes on a curvilinear grid that adapts to the geometry of the computational domain.

The results of numerical experiments allowed us to identify the most substantial characteristics of the phenomenon studied, investigate the peculiarities of the wave regime and their dependence on the slope of the coastal area, the distance from the wave generation zone, the presence or absence of protection structures, the relative importance of nonlinear and dispersion effects caused by abrupt changes in the velocity of the 'landslide' in the beginning and in the end of its motion. Some of the results obtained are given in the paper.

The paper consists of three sections. In Section 1, we consider the mathematical models and computational algorithms. Section 2 describes the problem; in Section 3 we discuss the results.

1. MATHEMATICAL MODELS AND COMPUTATIONAL ALGORITHMS

Linear, nonlinear, and nonlinear dispersion systems of shallow water equations generalized for the case of a nonstationary bottom surface are used as mathematical models of free surface dynamics of a heavy fluid [2]. We consider the case of a single spatial variable.

The nonlinear shallow water equations were written in the divergent form, which ensured the use of the algorithms constructed for the simulation of nearly discontinuous flows on the 'water-land' interface and the adequate reproduction of the motion of the shore line. Such wave regimes are caused by tsunami waves running up the beach and the motion of sliding masses along the bottom in one of the above models [7].

Thus, for the basic model we consider the equations

$$\begin{aligned} h_t + (hu)_x &= 0 \\ (hu)_t + \left(hu^2 + \frac{gh^2}{2} \right)_x &= ghH_x \end{aligned} \tag{1.1}$$

where u is the averaged flow velocity, h is the total fluid depth: $h = \eta + H$, η is the free surface elevation, H is the depth of the channel with undisturbed fluid. All the above quantities are functions of the variables x, t. The depth H is represented as $H(x,t) = \tilde{H}(x) - b(x,t)$, with $b(x,t) = 0$ for $t = 0$. The function $b(x,t) = 0$ describes the bottom surface dynamics and is supposed to be known.

The system of equations for the nonlinear dispersion model with the same variables has the form

$$\begin{aligned} h_t + (hu)_x &= \left[\tilde{H}\tilde{H}_x \left(\frac{1}{2}\tilde{H}_x u + \frac{1}{6}\tilde{H}u_x - \frac{1}{2}b_t \right) \right]_x \\ u_t + uu_x + g\eta_x &= \left[\tilde{H} \left(\frac{1}{3}\tilde{H}u_{tx} + \frac{1}{2}\tilde{H}_x u_t - \frac{1}{2}b_{tt} \right) \right]_x . \end{aligned} \tag{1.2}$$

The terms on the right-hand sides of these equations describe the dispersion and are of the order $O(H_0/l_0)^2$, where H_0 and l_0 are the typical depth and horizontal size.

The known McCormack scheme was used to approximate system of equations (1.1). The difference scheme studied in [4] was modified for nonlinear dispersion equations (1.2). Both schemes are of the second-order approximation. A smoothing procedure was used to eliminate nonphysical high-frequency oscillations.

The impermeability condition was given on the left boundary of the domain corresponding to a vertical wall. The non-reflection boundary condition [13] ensuring free wave outflow was given on the right boundary. At the initial instant for $t = 0$, the state of rest with an undisturbed free boundary was specified.

The simulation of the surface wave generation process by a model landslide shaped as a semi-ellipse was also based on the hydrodynamic model of potential plane-parallel flows. The two-dimensional area filled with the fluid was bounded from below by a partly mobile impermeable bottom. The impermeability condition was given on this part of the boundary, and the coincidence of the normal vector components of the fluid velocity and the moving bottom velocity was specified for the mobile bottom fragment. The fluid was bounded by a free surface from above. The mathematical formulation of the problem is to determine the velocity potential satisfying the Laplace equation and the functions describing the free boundary on which the kinematic and dynamic conditions have to be satisfied. In calculations we used moving grids, therefore, the derivation of finite difference equations was based on the approximation of equations written in the moving curvilinear coordinate system. In calculations we used the simplest grids that adapt only to the domain

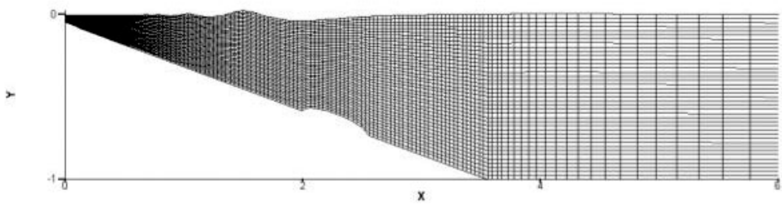


Figure 1. Computational grid for simulation by the full hydrodynamic model.

boundaries, including the moving ones. The second family of coordinate lines consisted of stationary vertical rectilinear segments uniformly spaced along the slope and rarefied by the law of geometric progression in the part of the basin that had the horizontal bottom (see Fig. 1). The nodes of the moving grid could move only in the vertical direction along the coordinate lines.

In the calculations we used a stepwise numerical algorithm in which at each time layer we first calculated the new values of the potential on the free boundary by the dynamic condition, which were then used as the Dirichlet boundary condition for the calculation of the potential in the interior of the domain, which satisfies the finite difference analogue of the Laplace equation. Using the obtained values of the potential, we specified the new location of the free boundary for the given time layer and constructed a grid for the successive time layer. A detailed description of the algorithm and the form of the finite difference equations on a moving curvilinear grid are given in [12].

2. DESCRIPTION OF MODEL PROBLEM

Below we consider the problem of studying wave regimes induced by the motion of a submerged solid body down a flat coastal slope. The shape of the solid body imitating a landslide is semi-elliptical. The major and minor semi-axes of this ellipse are 25 cm and 5 cm, respectively. The left boundary of the basin at a distance of x_g from the 'vertex' of the 'landslide' is a vertical impermeable wall with the abscissa $x = 0$. The water depth near this wall is assumed to be small. The basin in its right part has a horizontal bottom with a water depth of 90 cm (see Fig. 2).

We considered the uniformly accelerated motion of the model landslide with zero initial velocity. At a preset instant the 'landslide' instantly stopped, which always occurred on the sloping part of the bottom. The calculations continued long beyond the time period of the 'landslide' motion. The angle of the underwater slope varied in wide limits.

The variation of the free surface in time was registered at four points. The first point was located over the vertex of the semi-ellipse and had the abscissa x_g, the three other points were at a distance of $lm = 50$ cm from one another. The table gives the initial data for the numerical experiments whose results were compared with the data of the laboratory experiments [3].

Here φ is the slope angle, H_W is the water depth near the left wall, x_g is the abscissa of the vertex of the semi-ellipse at the initial instant $t = 0$, a is the 'landslide'

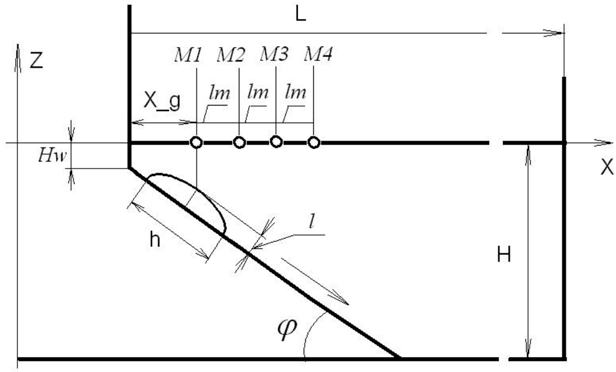


Figure 2. Scheme of laboratory experiment and computational domain.

Table 1.

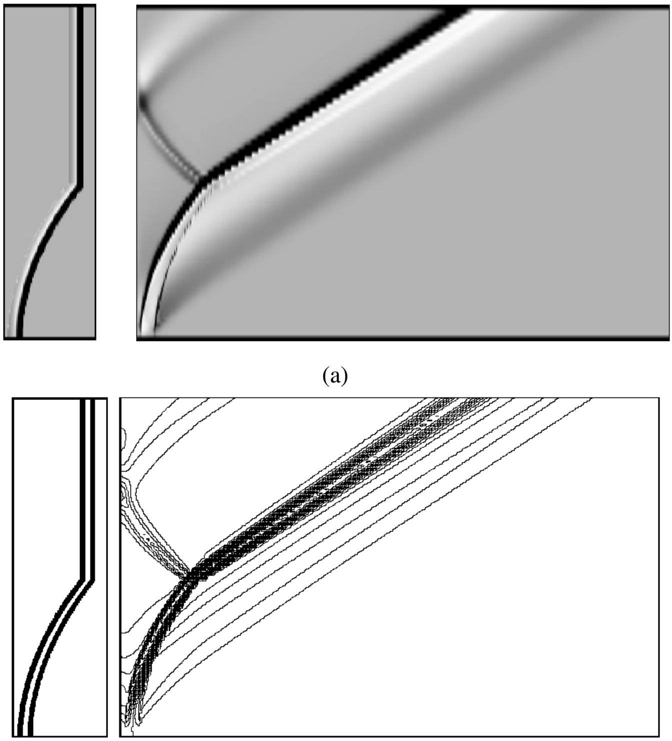
φ	H_W	x_g	a	T	S
10	0.048	0.66	0.6	3.3	3.27
15	0.043	0.40	0.6	2.7	2.19
30	0.046	0.28	1.3	1.3	1.10
45	0.040	0.27	2.49	0.7	0.61

acceleration, T is the time of its motion down the slope, S is the distance travelled down the slope. All linear sizes are given in metres, the time in seconds, the angles in degrees.

3. SIMULATION RESULTS

The qualitative characteristics of the simulated wave process are presented in Figs. 3 – 5. Each of these figures consists of two groups of patterns. The fragments (a) show the relief patterns of the process dynamics: the time varies vertically in the positive direction (from 0 to six seconds) and the distance along the basin varies horizontally. The fragments (b) give the same information as isolines of the free boundary levels. In each of the fragments on Figs. 3 – 5 the left parts represent the landslide motion and the right parts indicate the wave regime generated by this motion. Figure 6 illustrates the transformation of waves in their motion. All the figures show results obtained for a slope of 15 degrees. Other data are not included in this paper for lack of space.

The calculations by the linear and nonlinear shallow water models showed that the nonlinearity effect for the version considered (a slope of 15 degree) is very small, so that the results obtained by the linear and nonlinear models are essentially identical. Figure 3 shows that at the onset of the landslide motion, an elevation wave forms on the water surface propagating towards a larger depth, its length gradually increases until the shape of this wave approaches the shape of landslide.



(a)

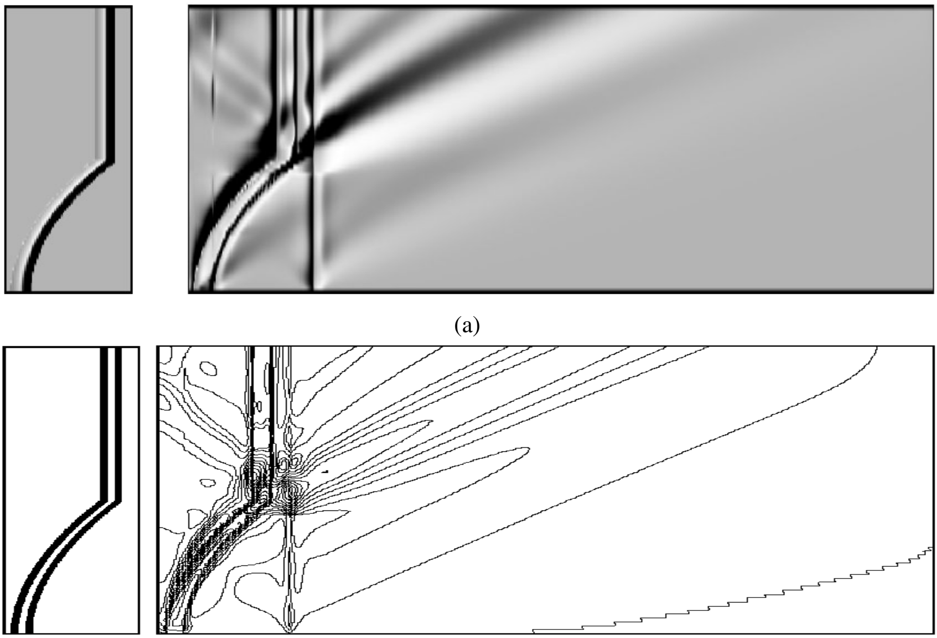
(b)

Figure 3. Computational results by the nonlinear shallow water model.

The wave amplitude practically does not change from one mareograph to another (see Fig. 6). As the slope angle increases, the wave amplitude and the wavelength somewhat decrease and, conversely, they increase when the slope angle decreases. The rising wave is followed by a falling one with a much larger amplitude. When the falling wave propagates, its amplitude increases and in the fourth mareograph reaches (Fig. 6d) a value almost three times larger than that in the first mareograph (Fig. 6a). In varying the slope angle, the above trends persist.

After the landslide stops, a rising wave propagating towards the coast is formed. The amplitude of this wave slightly increases in motion but the wavelength remains virtually unchanged. In Fig. 3 the above effect is represented as a specific 'triple' configuration. After the landslide stops, the deep-water wave passes to the horizontal bottom zone, which is accompanied by a change in the velocity of this wave. We also note the effect of the reflection of the shallow water wave from the wall and the occurrence of a rising wave moving to a larger depth. As the wave moves over the slope, its amplitude decreases and the depression following the wave flattens.

We should emphasize some features of the wave regime calculated by the nonlinear dispersion model (Fig. 4), in particular, the strong sensitivity of the mathematical model to input data smoothness, which is illustrated by the distortion of the free surface profile at the point of the depth gradient variation, i.e., the point



(a)

(b)

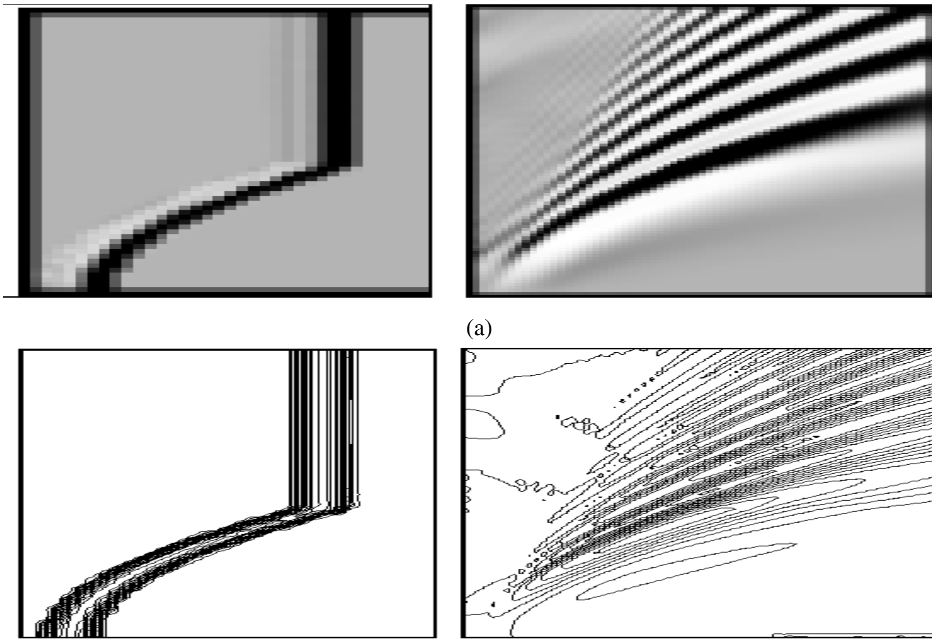
Figure 4. Computational results by the nonlinear dispersion shallow water model.

of contact of the slope zone with the horizontal bottom zone. Under the dispersion effect the number of waves increases, the free surface rearrangement at the moment of the landslide stop becomes more complicated, the amplitude of the waves propagating towards the shallow water decreases. The above effects may be caused by a more accurate account of the vertical processes in the nonlinear dispersion model. The validity of this explanation may be confirmed or refuted by comparison with the computational results based on the full (vertically two-dimensional) model and with experimental data.

The results shown in Fig. 5 confirm, to some extent, the above assumption. They demonstrate a sufficiently complex wave pattern in which the landslide motion generates a series of waves propagating to the deep-water zone. The above components of the wave regime occur, but they are less pronounced than those in the linear and nonlinear formulations of the shallow water theory. Note that these calculations were carried out in a smaller computational domain because of the large bulk of calculations.

The deciding argument can be obtained from a comparison with experimental data. For this purpose we should turn to Fig. 6, its fragments show mareograms measured in a laboratory experiment and calculated by all the models at each mareograph points for a slope angle of 15 degrees. Note that all the graphs coincide in their first absolute minimum.

According to the mareograms, the poorest coincidence is observed in the first mareograph located immediately above the center of the initial position of the land-



(a)

(b)

Figure 5. Computational results by the model of potential fluid flows with free surface.

slide. In the subsequent mareograms all the models used quite adequately reproduce the form of the first oscillations, the nonlinear equations of shallow water theory leading to a considerable amplitude excess. The linear model proves sufficiently close to the experimental data at the initial stage of the process. However, later the vertically averaged equations result in a simplified wave regime, rather dissimilar to that obtained in the experiment. The above mentioned individual elements of the wave regime are quite evident, i.e., the leading rising wave, the subsequent considerable decrease in the level, the wave occurring at the instant the landslide stops, and the wave reflected from the coast.

Finally, the equations of the full hydrodynamic model reproduce not only the frequency variations of the wave regime, but also allow us to calculate the most accurate values of the amplitudes in the whole wave train, which is a serious argument for taking into account the vertical effects when modelling the landslide mechanism of surface wave generation.

The next series of patterns (Fig. 7) actually demonstrates the vertical structure (streamlines) of the flow studied. Over the initial period of the landslide motion (Fig. 7a) a ‘condensation’ zone periodically occurs in front of it rolling over the moving obstacle; a decrease in the level on the water surface occurs immediately behind the landslide, in the near-surface layer the streamlines assume a characteristic arcwise form. Before the landslide stops (Fig. 7b), these structures are finally formed. At the moment of its stop (Fig. 7c) the streamlines are pressed to the landslide transforming the final stage of the process to flowing around a stationary body

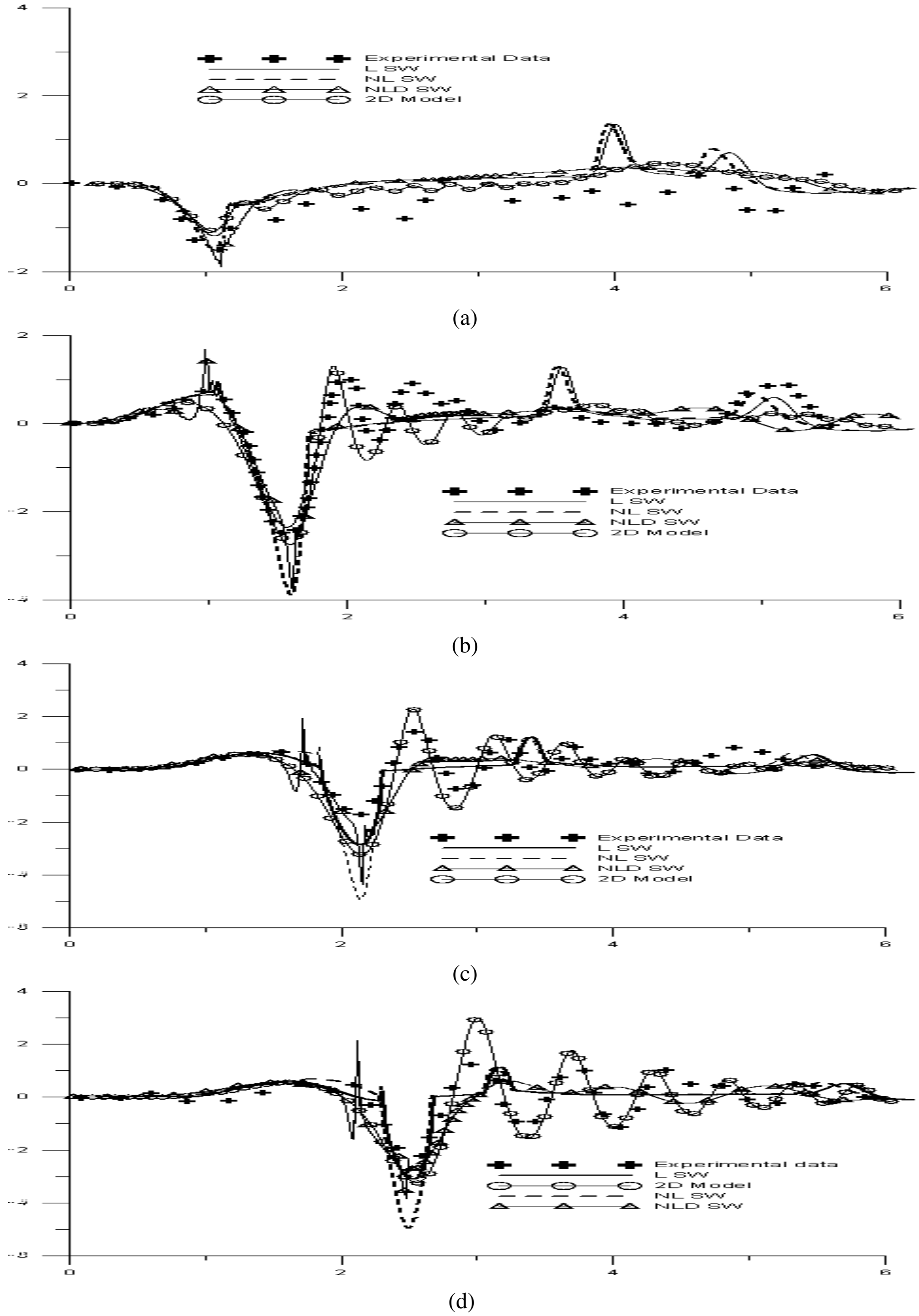


Figure 6. Experimental and calculated data in four successively located mareographs, a slope of 15 degrees.

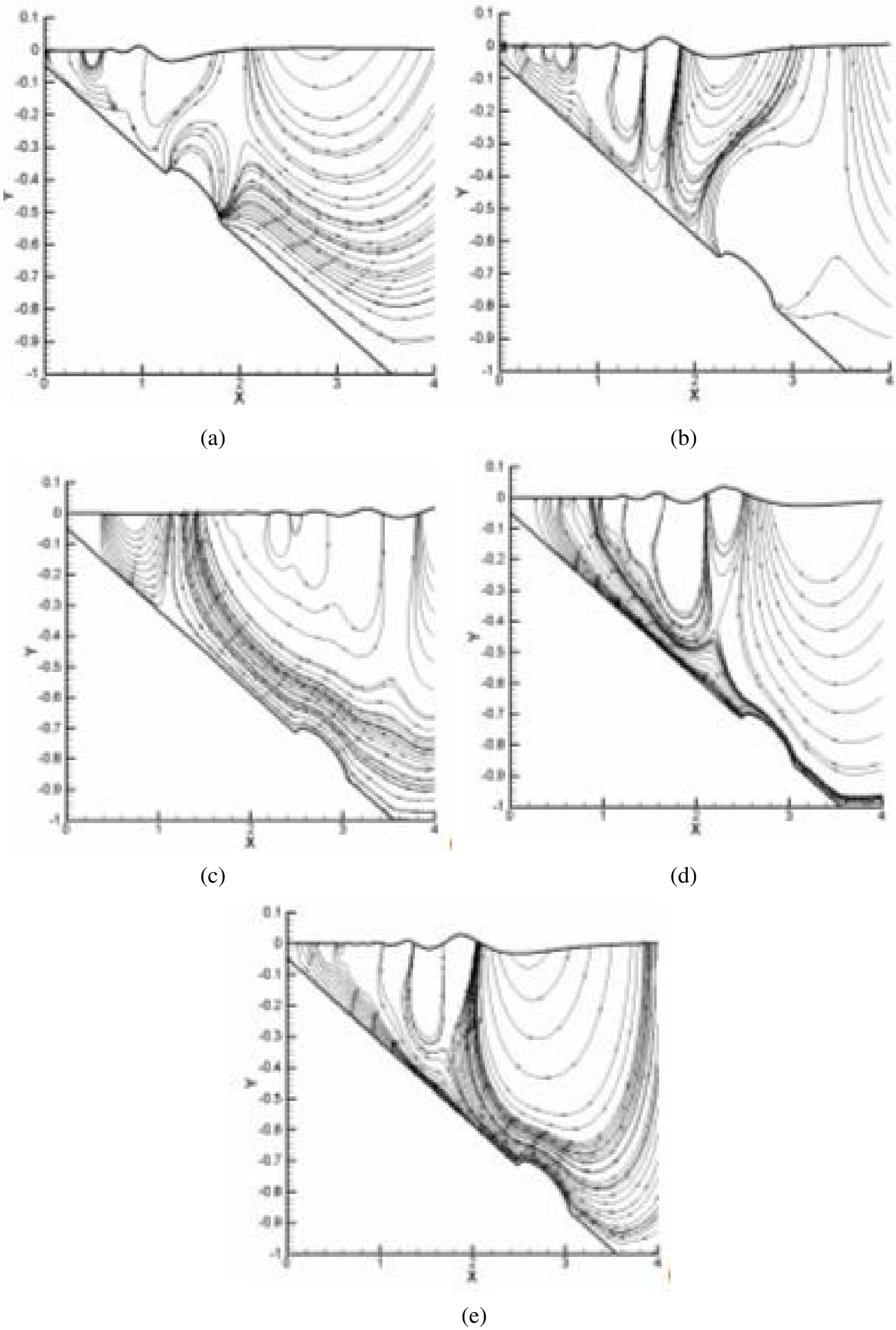


Figure 7. Streamlines: (a) – at the initial stage of the process, (b) – before the landslide stop, (c) – the instant of the landslide stop, (d) – immediately after the landslide stop, (e) – at the final stage of calculation.

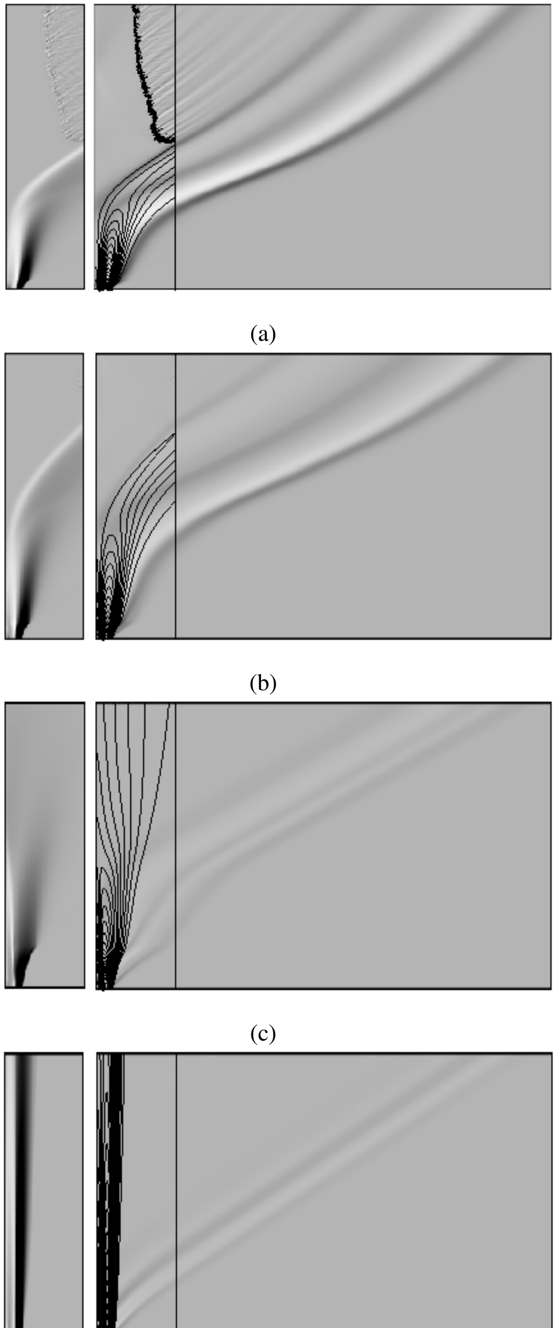


Figure 8. Computational results using shallow water theory for describing landslide motion: (a) – without friction, (b) – with low friction, (c) – with moderate friction, (d) – with intense friction.

(Figs. 7d and 7e). In this case wedge-shaped ‘condensation’ zones arise as well (see Fig. 7d) and roll over the landslide towards the larger depth. Note that some distortions result from the variations in the grid structure in the zone of the transition from the slope to the constant-depth bottom.

4. CONCLUSION

The results of the laboratory and numerical experiments given in the work suggest the possibility of using approximate mathematical models for describing the initial stage of the surface waves generation by the landslide mechanism and point to the fundamental importance of the assessment of the vertical processes that, in essence, specify the wave field structure.

We should consider the new nonlinear dispersion models with an improved dispersion relation and continue a comprehensive study using rather time-consuming full hydrodynamic models. It is also necessary to place primary emphasis upon the initial data smoothness. As to the computational algorithms, particular attention should be given to the computational grid adaptation to the changing bottom shape and methods for constructing the efficient high-precision schemes for approximating equations with higher derivatives.

There are new prospects in the simulation of the motion of sliding masses which, to a considerable extent, specifies the character of the wave regime. Figure 9 presents, as an example, the results obtained by the shallow water model and for the description of the motion of a landslide regarded as a double density fluid moving down a slope of 15 degrees with a given friction. This formulation of the problem is considered in greater detail in [1]. As seen from this series of patterns, the change in the landslide motion model fully specifies the wave regime on the free surface, which is still of a simplified character. As the friction increases, the distance the landslide travels decreases and its spreading reduces. For a low level of friction the landslide at some moment starts moving towards a larger depth, its motion down the slope accelerated. The wave at the initial stage fully follows the trajectory of the landslide and after its stops continues to move to the constant-depth area. Its amplitude considerably decreases with increasing friction. As the friction increases, the landslide remains practically stationary, whereas a weak perturbation occurring at the start of the motion propagates along a constant trajectory.

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